

Unparticle decay of neutrinos and it's effect on ultra high energy neutrinos

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Abstract

The unparticle is proposed by Georgi as a conceptual new physics beyond the standard model to describe the low energy sector of a nontrivial scale invariant sector of an effective theory. We consider the neutrino decay in unparticle scenario and investigate the effect of such decay for the case of ultra high energy neutrinos from GRB. The effect on the detection yield of these neutrinos are also probed for a kilometer scale ice-Cerenkov detector.

1 Introduction

The scale invariance is a feature in which a theory or a law does not change if the energy scales or length scales are given a multiplicative transformation by a common factor. The transformation is also known as dilatation and this has applications in both mathematics and physics. In mathematics, the scale invariance can be described by self similarity. An object is such that if it is magnified by any amount, a smaller piece of the object always resembles the whole object. Fractals can be a typical example. In statistical mechanics, a scale invariant theory is needed to explain the phenomenon of phase transition as the fluctuations near the critical point become scale invariant. The scale invariance can apply in classical field theory, string theory etc. In quantum field theory, the scale invariance, can very naively be interpreted as the non-dependence of the particle interaction strength on the energy of the particles.

In the real world, the scale invariance is manifestly broken by the masses of the standard model particles. The scale transformation involves mass dimension and therefore in scale invariant scenario, the particle masses have to be zero. Therefore standard model does not have scale invariance. At a scale much higher than that of standard model, the scale invariance is restored. In a recent work [1, 2] Georgi observed that if an yet unseen scale invariant sector is present in four dimensions and very weakly interacting with the standard model particles, then this scale invariant sector contains not “particles” but massless “Unparticles”.

Under the framework of field theory, an interacting scale invariant theory is invariant under conformal group transformation. A conformal group transformation is a simple transformation and can be generated (in Minkowsky space) by making a scale transformation of the type $x^\mu \rightarrow \lambda x^\mu$ (μ being the space-time coordinates) [3]. Although conformal invariance is well studied in two dimensional scenario, it is rare in four dimensional quantum field theory. The renormalization group effects break the conformal invariance [4].

In field theory, renormalization group flows from some scale invariant ultra violet fixed points to some scale invariant infra red fixed points. However, this exceptional scenario of conformal invariance in four dimension can be described by the vector-like non abelian gauge theory with suitable number of massless fermions. Such a theory is first studied by Banks and Zaks (\mathcal{BZ}) [5]. The theory has non-trivial infra red fixed points. At low energy limits a non-trivial conformal sector is ensured by the presence of non-trivial infra red fixed points in the theory. But this requires the number of fermion generations to be non-integral number and therefore is not manifested in nature.

As suggested in Ref. [1, 2] such a nontrivial scale invariant sector due to \mathcal{BZ} field might appear at TeV scale. The \mathcal{BZ} field interact with the standard model field via the exchange of a particle with large mass scale $M_{\mathcal{U}}$. This can be termed as connector sector [6]. Below $M_{\mathcal{U}}$ standard model and \mathcal{BZ} field couplings are suppressed by powers of $M_{\mathcal{U}}$. Generically the operators can be written as [1], $\mathcal{O}_{\text{SM}}\mathcal{O}_{\mathcal{BZ}}/M_{\mathcal{U}}^k$, ($k > 0$) where \mathcal{O}_{SM} and $\mathcal{O}_{\mathcal{BZ}}$ are operators of mass dimensions d_{SM} and $d_{\mathcal{BZ}}$ respectively and constructed from standard model and \mathcal{BZ} fields respectively. Below the connector sector such interactions between \mathcal{BZ} and standard fields can be parametrized in terms of non-renormalizable interactions and the suppression of nonrenormalizable operators by powers of $M_{\mathcal{U}}^k$ are induced. As the scale invariance sets in at an energy scale $\Lambda_{\mathcal{U}}$, the renormalizable couplings of the \mathcal{BZ} fields in the scale invariant sector induce dimensional transmutation [7] at $\Lambda_{\mathcal{U}}$. The particles in this sector is then described by massless unparticles [1]. In the effective theory, below the scale $\Lambda_{\mathcal{U}}$, the \mathcal{BZ} fields match onto the unparticle operators and a new set of operators having the form $(C_{\mathcal{O}_{\mathcal{U}}}\Lambda^{d_{\mathcal{BZ}}-d_{\mathcal{U}}}/M_{\mathcal{U}}^k)\mathcal{O}_{\text{SM}}\mathcal{O}_{\mathcal{BZ}}$, where $d_{\mathcal{U}}$ is the scale dimensions of unparticle operator $\mathcal{O}_{\mathcal{U}}$ and $C_{\mathcal{O}_{\mathcal{U}}}$ is a coefficient to be fixed by the matching. This may be occurred that the coupling between the unparticle and standard model particles may violate the conformal invariance but in [2] it is argued that the infrared fixed points of the unparticles will not be affected by such couplings since \mathcal{BZ} fields decou-

ple from the standard model particles at low energies. As observed in Ref. [1], the unparticle stuff with scale dimension d_U appears to be a nonintegral number d_U of massless particles.

The unparticle physics opens up the possibility of very unexpected phenomenology and the effects of unparticle coupling for specific processes are studied by various authors [8]. Kikuchi and Okada [9] addressed the Higgs phenomenology with unparticles where they considered operators involving scalar unparticle, Higgs and the gauge bosons (gluons and photons) and discussed the effective couplings between the Higgs boson and the gauge bosons. The interaction of unparticles with Standard Model particles is also addressed by Chen and He [10]. Recently Zhou [11] discussed the possibility of neutrino decaying into unparticle. Neutrino decay to unparticles are also discussed by Chen et al in Ref. [12]. If such neutrino decay really occurs it may have consequences in several ongoing neutrino experiments as also the future experiments. In a recent work Li et al [13] has discussed the neutrino decay in unparticle scenario as a possible explanation of the excess electron like events observed by the MiniBooNe [14] neutrino experiment. In the present work, the effect of such neutrino decay to unparticles is considered on ultra high energy (UHE) neutrinos arriving on earth from distant cosmic sources such as gamma ray bursts (GRBs). The decay length for decays to unparticles depends on quantities like the unparticle-neutrino coupling, the unparticle scale factor etc. The decay length for this scenario can be large enough (\sim tens of Mpc) and thus can significantly affect the survival probability of a decaying neutrino if it traverses a baseline length of Mpc order. The ultra high energy neutrinos from distant GRBs at astronomical distances from the earth may indeed provide such a long baseline. Also, for such a long baseline the oscillatory effect of neutrinos are averaged out and therefore opens up the possibility to effectively probe such decays, if any, in signals from UHE neutrinos at terrestrial detectors such as IceCube [15].

The paper is organised as follows. In Section 2, the formalism is discussed. This includes the neutrino flux from a GRB at a redshift z ; the

survival probability of such neutrinos on reaching the earth if they undergo decay to unparticles. Therefore the actual flux of neutrinos on reaching the earth is obtained from folding the original flux with the survival probability. These are described in Section 2.1. In Section 2.2, the analytical expressions for the yield of secondary muons and shower events induced by the UHE neutrinos at the ice Cerenkov detector are described. The calculational procedure of the yield at the kilometer scale ice Cerenkov detector such as IceCube for the neutrino flux described in Section 2.1, is given in Section 3. calculational results are also discussed in this section. Finally, Section 4 contains discussions and summary.

2 Formalism

2.1 GRB neutrino flux with neutrino decay to unparticles

The GRB neutrino flux for a particular redshift is estimated considering the the relativistic fireball model [16]. In this type of GRB model, protons (also electrons, positrons and photons) produced in the magnetic field of the rotating accretion disc around a possible black hole are accelerated perpendicular to the accretion disc at almost the speed of light. This forms a jet which is referred to as fireball. The burst is supposed to be the dissipation of kinetic energy of this relativistic expanding fireball. High energy pions are photoproduced from Δ resonance when the protons in the jet interacts with photons. These pions then decay to yield ν_μ and ν_e in the approximate proportion of 2:1.

The neutrino flux from a GRB depends on several GRB parameters like Lorentz boost factor Γ (required for the transformation from the fireball blob to observer's frame of reference), the photon break energy (as the photon spectrum is considered broken) and the photon luminosity L_γ (generally \sim

10^{53} ergs/sec).

With all these, the neutrino spectrum from a GRB can be parametrised as [17, 18]

$$\frac{dN_\nu}{dE_\nu} = A \times \min(1, E_\nu/E_\nu^b) \times \frac{1}{E_\nu^2}. \quad (1)$$

In the above, E_ν is the neutrino energy N_ν is the number of neutrinos and

$$\begin{aligned} E_\nu^b &\simeq 10^6 \frac{\Gamma_{2.5}^2}{E_{\gamma, \text{MeV}}^b} \text{GeV} \\ \Gamma_{2.5} &= \Gamma/10^{2.5} \\ A &= \frac{E_{\text{GRB}}}{1 + \ln(E_{\nu\text{max}}/E_\nu^b)}, \end{aligned} \quad (2)$$

where $E_{\nu\text{max}}$ is the cut-off energy for the GRB neutrinos and E_{GRB} is the total energy that a GRB emits. Now, the observed energy E_ν^{obs} of a neutrino with the actual energy E_ν coming from a GRB at a redshift distance z is given by the relation $E_\nu^{\text{obs}} = E_\nu/(1+z)$ and similarly, the maximum observable neutrino energy $E_{\nu\text{max}}^{\text{obs}}$ is $E_{\nu\text{max}}^{\text{obs}} = E_{\nu\text{max}}/(1+z)$. The comoving distance d of a GRB at redshift z is given by

$$d(z) = \frac{c}{H} \int_0^z \frac{dz'}{\sqrt{\Omega_\Lambda + \Omega_M((1+z')^3)}} \quad (3)$$

where Ω_M is the matter density of the universe at present epoch, Ω_Λ is the dark energy density respectively in units of critical density of the universe and c, H are the velocity of light in vacuum and Hubble constant respectively. In the present calculation $c = 3 \times 10^5$ Km/sec and $H = 72$ Km/sec/Mpc (1 Mpc = 3.086×10^{19} Km). Therefore the neutrinos from a single GRB that can be observed on earth per unit energy per unit area of the earth is given by,

$$\frac{dN_\nu^{\text{obs}}}{dE_\nu^{\text{obs}}} = \frac{dN_\nu}{dE_\nu} \frac{1}{4\pi d^2(z)} (1+z) \quad (4)$$

The production process of UHE neutrinos suggests that the neutrino flavours are produced in the ratio $\nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0$. In case of

mass flavour oscillation, because of the astronomical baseline ($L \sim \text{Mpc}$), the acquired relative phases of the propagating neutrino mass eigenstates is averaged out ($\Delta m^2 L/E \gg 1$) and the UHE neutrinos from a GRB reaching the earth are incoherent mixture of mass eigenstates. In fact, it can be shown [19] that for $\theta_{23} = 45^\circ$ (maximal mixing) and $\theta_{13} \simeq 0$, the flavour ratio on reaching the earth, for neutrino mass-flavour oscillation with such a long astronomical baseline, becomes $\nu_e : \nu_\mu : \nu_\tau = 1 : 1 : 1$ irrespective of the solar mixing angle.

However, in the present work we consider neutrino decay to the recently proposed “Unparticle” by Georgi and investigate that in the event of such decay, the possible signal of UHE neutrinos from distant GRBs in terms of muon and shower yields of a kilometer scale detector such as IceCube.

We consider here a decay hypothesis where a neutrino mass eigen state ν_j decays to an unparticle and another neutrino mass eigenstate ν_i following the decay relation

$$\nu_j \rightarrow \mathcal{U} + \nu_i$$

The unparticle physics proposed by Georgi [1] indicates a possibility for neutrino decaying into the so called unparticle [11]. The effective Lagrangian for the process can be written as

$$\mathcal{L} = \frac{\lambda_\nu^{\alpha\beta}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{\nu}_\alpha \gamma_\mu \nu_\beta \mathcal{O}_{\mathcal{U}} \quad (5)$$

where $\mathcal{O}_{\mathcal{U}}$ is the unparticle operator, λ_ν is the relevant coupling constant and α, β are the flavour indices. In Eq. (7) dimension transmutation scale is $\Lambda_{\mathcal{U}}$ at which the scale invariance sets in and $d_{\mathcal{U}}$ is the scaling dimension.

In the mass basis, the interaction term (between neutrino and unparticle) can be written as

$$\frac{\lambda_\nu^{ij}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{\nu}_i \gamma_\mu \nu_j \mathcal{O}_{\mathcal{U}} \quad (6)$$

where i, j are the mass eigenstate indices and

$$\lambda_\nu^{ij} = \sum_{\alpha, \beta} U_{\alpha i}^* \lambda_\nu^{\alpha\beta} U_{\beta j} \quad (7)$$

In the above U , is the Maki-Nakagawa-Sakata (MNS) mixing matrix (mass-flavour) for neutrinos. The decay rate for the process $\nu_j \rightarrow \nu_i + \mathcal{U}$ (\mathcal{U} is the unparticle) can now be elected [11] and the neutrino lifetime τ for such decay process is given as

$$\frac{\tau_{\mathcal{U}}}{m_j} = \frac{2_{\mathcal{U}}^d \pi^2 d_{\mathcal{U}} (2 - d_{\mathcal{U}}) (d_{\mathcal{U}} + 1)}{3 A_{d_{\mathcal{U}}} |\lambda_{\nu}^{ij}|^2} \left(\frac{\Lambda_{\mathcal{U}}^2}{m_j^2} \right)^{d_{\mathcal{U}}-1} \frac{1}{m_j} . \quad (8)$$

In Eq. (10) above, m_j is the mass of the decaying neutrino and $A_{d_{\mathcal{U}}}$ is a normalization constant given by [1]

$$A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}} \frac{\Gamma(d_{\mathcal{U}} + 1/2)}{\Gamma(d_{\mathcal{U}} - 1)\Gamma(2d_{\mathcal{U}})} \quad (9)$$

A decay scenario can be one in which both the states $|\nu_2\rangle$ and $|\nu_3\rangle$ are unstable and decay whereas the lightest state $|\nu_1\rangle$ is stable. Needless to say that the normal mass hierarchy is implicit in this scenario. Following [23], for this decay scheme with the condition that the coherence is lost (baseline $L \gg 1/\Delta m^2$ and the oscillatory part is absent), the flux of ν_a for flavour a on reaching the earth from a distant GRB at a distance L from the earth, is written as

$$\phi_{\nu_a}(E) = \sum_b \sum_i \phi_{\nu_b}^s(E) |U_{bi}|^2 |U_{ai}|^2 \exp(-4\pi L/(\lambda_d)_i) . \quad (10)$$

In the above, a, b are the flavour indices, i is the mass index and U is the MNS mixing matrix for neutrinos. The exponential term in the above equation is the decay term with $(\lambda_d)_i$ is the decay length given by,

$$(\lambda_d)_i = 2.5km \frac{E}{\text{GeV}} \frac{\text{eV}^2}{\alpha_i} \quad (11)$$

where E is the neutrino energy and $\alpha_i (= m_i/\tau)$, τ being the rest frame decay lifetime and m_i is the mass of the i^{th} decaying neutrino. Thus, the decay lifetime in lab frame ($\sim E\tau/m$) and hence the decay length λ_d has a strong dependence on the neutrino energy too.

The fluxes of ν_e , ν_μ and ν_τ from a distant GRB can now be calculated for this decay scenario using Eqs (1-4) and Eqs (10-11). Eq. (4) gives the total flux in absence of any oscillation or decay. Considering the fact that at source the flux ratio of ν_e , ν_μ and ν_τ is 1:2:0, in absence of decay or oscillation, the ν_e flux ($\phi_{\nu_e}^s$) and ν_μ flux ($\phi_{\nu_\mu}^s$) and ν_τ flux ($\phi_{\nu_\tau}^s$) can be written in terms of $\phi_\nu = \frac{dN_\nu^{\text{obs}}}{dE_\nu^{\text{obs}}}$,

$$\phi_{\nu_e}^s = \frac{1}{3}\phi_\nu, \quad \phi_{\nu_\mu}^s = \frac{2}{3}\phi_\nu = 2\phi_{\nu_e}^s, \quad \phi_{\nu_\tau}^s = 0 \quad (12)$$

Applying the above equation for ν_e flux ϕ_{ν_e} on arrival at earth and with the condition that only $|\nu_1\rangle$ is stable, we get,

$$\begin{aligned} \phi_{\nu_e} &= \phi_{\nu_e}^s(E)|U_{e1}|^2|U_{e1}|^2 + 2\phi_{\nu_e}^s(E)|U_{\mu1}|^2|U_{e1}|^2 + \\ &\quad \phi_{\nu_e}^s(E)|U_{e2}|^2|U_{e2}|^2 \exp(-4\pi L/(\lambda_d)_2) + \\ &\quad 2\phi_{\nu_e}^s(E)|U_{\mu2}|^2|U_{e2}|^2 \exp(-4\pi L/(\lambda_d)_2) + \\ &\quad \phi_{\nu_e}^s(E)|U_{e3}|^2|U_{e3}|^2 \exp(-4\pi L/(\lambda_d)_3) + \\ &\quad 2\phi_{\nu_e}^s(E)|U_{\mu3}|^2|U_{e3}|^2 \exp(-4\pi L/(\lambda_d)_3). \end{aligned} \quad (13)$$

The first two terms on RHS of Eq. (13) (the terms without decay factor) can be written as

$$\phi_{\nu_e}^s |U_{e1}|^2 [|U_{e1}|^2 + 2|U_{\mu1}|^2] = |U_{e1}|^2 [1 + |U_{\mu1}|^2 - |U_{\tau1}|^2] \quad (14)$$

where the use has been made of the unitarity condition $\sum_i U_{ai}U_{bi} = \delta_{ab}$. For $\theta_{13} = 0$ ($U_{e3} \simeq 0$), $|U_{\mu j}|^2 - |U_{\tau j}|^2 \simeq 0$. The other terms of Eq. (17) can also thus be simplified for $\theta_{13} \simeq 0$. With similar approach for ν_μ and ν_τ fluxes (ϕ_{ν_μ} and ϕ_{ν_τ} respectively) can be calculated. The expressions for three neutrino fluxes on arrival at the earth are simplified as (in terms of the source flux)

$$\begin{aligned} \phi_{\nu_e} &= \phi_{\nu_e}^s(E)[|U_{e1}|^2 + |U_{e2}|^2 \exp(-4\pi L/(\lambda_d)_2) \\ &\quad + |U_{e3}|^2 \exp(-4\pi L/(\lambda_d)_3)] \end{aligned}$$

$$\phi_{\nu_\mu} = \phi_{\nu_e}^s(E)[|U_{\mu1}|^2 + |U_{\mu2}|^2 \exp(-4\pi L/(\lambda_d)_2)]$$

$$\begin{aligned}
& + |U_{\mu 3}|^2 \exp(-4\pi L/(\lambda_d)_3) \\
\phi_{\nu_\tau} &= \phi_{\nu_e}^s(E)[|U_{\tau 1}|^2 + |U_{\tau 2}|^2 \exp(-4\pi L/(\lambda_d)_2) \\
& + |U_{\tau 3}|^2 \exp(-4\pi L/(\lambda_d)_3)]
\end{aligned} \tag{15}$$

From Eqs. (10,13,15) one can see that in case of $L \gg \lambda_d$, the decay effect is washed out. The decay lifetime (τ) becomes so small that the neutrino under consideration decays completely much before it reaches the earth. Under this condition the flavour ratio takes the form $\phi_{\nu_e} : \phi_{\nu_\mu} : \phi_{\nu_\tau} = |U_{e1}|^2 : |U_{e2}|^2 : |U_{e3}|^2$ (from Eq. 15) as discussed in [23]. In this case, the actual distance of UHE neutrino sources such as GRB is not very important and one can work with the diffuse GRB neutrino flux such as given in Ref. [22]. But, if $\lambda_d \sim L$ then one cannot neglect the exponential term in Eqs. (13) and (15) and knowledge of L is essential. In such decay event, it is useful to work with neutrino fluxes from GRBs with definite redshifts (z) (and hence different baselines L) in order to probe the effects of neutrino decay. In the present work, oscillation parameters are chosen as $\theta_{23} = 45^\circ$, $\theta_{13} = 0$ and $\theta_{12} = 32.31^\circ$ and redshift $z = 0.03$ unless otherwise mentioned.

It may be noted here, that for a GRB with redshift $z = 0.03$ ($\sim 10^{21}$ km $= L$), the decay length $\lambda_d \simeq 4 \times 10^{20}$ km for unparticle coupling $\lambda = 0.004$, $d = 1.3$ and $\Lambda_{\mathcal{U}} = 1$ TeV at neutrino energy $E_\nu \simeq 2$ TeV. Therefore for neutrino decay into an unparticle, the neutrino decay length is indeed of the order of the baseline length L .

2.2 Detection of UHE neutrinos

The ultra high energy neutrinos from a GRB can be detected in a terrestrial detector such as IceCube by detecting the secondary products like muons, tauons, electromagnetic or hadronic showers that are produced due to charged current (CC) and neutral current (NC) interactions of UHE neutrinos with the terrestrial rock and detector material. The IceCube is a 1 km³

ice/water Cerenkov detector in south pole ice. The secondary muons that are produced via CC interaction can be detected by track signal. The ν_τ CC interactions produce the so called “Double Bang” events (track + shower) whereas ν_e CC interactions gives electromagnetic shower at the detector. In the present calculation however, we do not consider the “double bang” events as this is perhaps effective for a narrow energy range of 1 PeV to 20 PeV for kilometer cube detector [24]. For ν_τ CC interactions we consider instead the decay channel ($\tau \rightarrow \bar{\nu}_\mu \mu \nu_\tau$) of secondary tauons where the muons thus produced give track signals. The NC interactions of all neutrino flavours produce shower events.

The total number of secondary muons induced by GRB neutrinos at a detector of unit area is given by (following [25, 26, 17])

$$S = \int_{E_{\text{thr}}}^{E_{\nu}^{\text{obs}}} dE_\nu^{\text{obs}} \frac{dN_\nu^{\text{obs}}}{dE_\nu^{\text{obs}}} P_{\text{surv}}(E_\nu^{\text{obs}}, \theta_z) P_\mu(E_\nu^{\text{obs}}, E_{\text{thr}}), \quad (16)$$

where P_{surv} is the probability that a neutrino reaches the detector without being absorbed by the earth. This is a function of the neutrino-nucleon interaction length in the earth given by $L_{\text{int}} = \{\sigma^{\text{tot}}(E_\nu^{\text{obs}}) N_A\}^{-1}$, (N_A is the Avogadro number and σ^{tot} is the sum total of CC and NC interactions) and the effective path length $X(\theta_z)$ (gm cm^{-2}) for incident neutrino zenith angle θ_z . P_{surv} takes the form,

$$P_{\text{surv}}(E_\nu^{\text{obs}}, \theta_z) = \exp[-X(\theta_z)/L_{\text{int}}] = \exp[-X(\theta_z)\sigma^{\text{tot}} N_A].$$

In Eq. (16),

$$P_\mu(E_\nu^{\text{obs}}, E_{\text{thr}}) = N_A \sigma^{\text{CC}} \langle R(E_\nu^{\text{obs}}; E_{\text{thr}}) \rangle, \quad (17)$$

with N_A is the Avogadro number and σ^{CC} is the ν_μ CC interaction. The average range of muon inside the rock is given by

$$\langle R(E_\nu^{\text{obs}}; E_{\text{thr}}) \rangle = \frac{1}{\sigma^{\text{CC}}} \int_0^{1-E_{\text{thr}}/E_\nu} dy R(E_\nu^{\text{obs}}(1-y), E_{\text{thr}}) \frac{d\sigma^{\text{CC}}(E_\nu^{\text{obs}}, y)}{dy} \quad (18)$$

where $y = (E_\nu^{\text{obs}} - E_\mu)/E_\nu^{\text{obs}}$. The range $R(E_\mu, E_{\text{thr}})$ for a muon of energy E_μ is given as

$$R(E_\mu, E_{\text{thr}}) = \int_{E_{\text{thr}}}^{E_\mu} \frac{dE_\mu}{\langle dE_\mu/dX \rangle} \simeq \frac{1}{\beta} \ln \left(\frac{\alpha + \beta E_\mu}{\alpha + \beta E_{\text{thr}}} \right) \quad (19)$$

The average lepton energy loss with energy E_μ per unit distance travelled is given by [25]

$$\left\langle \frac{dE_\mu}{dX} \right\rangle = -\alpha - \beta E_\mu \quad (20)$$

The values of α and β used in the present calculations are

$$\begin{aligned} \alpha &= \{2.033 + 0.077 \ln[E_\mu(\text{GeV})]\} \times 10^{-3} \text{GeVcm}^2 \text{gm}^{-1} \\ \beta &= \{2.033 + 0.077 \ln[E_\mu(\text{GeV})]\} \times 10^{-6} \text{cm}^2 \text{gm}^{-1} \end{aligned} \quad (21)$$

for $E_\mu \lesssim 10^6$ GeV [27] and

$$\begin{aligned} \alpha &= 2.033 \times 10^{-3} \text{GeVcm}^2 \text{gm}^{-1} \\ \beta &= 3.9 \times 10^{-6} \text{cm}^2 \text{gm}^{-1} \end{aligned} \quad (22)$$

otherwise [28].

3 Calculations and Results

We define a ratio R of the track events and shower events at the kilometer scale detector considered here.

$$R = \frac{\text{Muon track events}}{\text{Shower events}} \quad (23)$$

The muon track events are therefore the total sum of the events from ν_μ CC interactions and ν_τ CC interactions (as described in the previous section). They are obtained using Eqs (16 - 22) with the flux in the expression for S (total number of secondary muons) of Eq. (16) suitably replaced by ϕ_{ν_μ} or ϕ_{ν_τ} (Eq. 15) as the case may be. The neutrino CC and NC cross-sections

(σ^{CC} and σ^{NC}) are taken from Ref. [22]. The effective path length used in the expression for P_{surv} (Eq. (16)) is written as $X(\theta_z) = \int \rho(r(\theta_z, \ell)) d\ell$ where $\rho(r(\theta_z, \ell))$ is the matter density inside the earth at a distance r from the centre of the earth and ℓ is the neutrino path length with zenith angle θ_z . The earth matter density is taken from Ref. [26] that follows from preliminary earth reference model (PREM). The calculations are made for GRB neutrino zenith angle $\theta_z = 100.9^\circ$.

The shower events are obtained using the equation

$$N_{\text{sh}} = \int dE_\nu \frac{dN_\nu}{dE_\nu} P_{\text{surv}}(E_\nu) \times \int \frac{1}{\sigma^j} \frac{d\sigma^j}{dy} P_{\text{int}}(E_\nu, y) , \quad (24)$$

where, $\sigma^j = \sigma^{CC}$ (for electromagnetic shower from ν_e charged current interactions) or σ^{NC} as the case may be and P_{int} is the probability that a shower produced by the neutrino interactions will be detected and is given by

$$P_{\text{sh}} = \rho N_A \sigma^j L \quad (25)$$

where ρ is the density of the detector material and L is the length of the detector ($L = 1$ Km for IceCube). Here too, the flux dN_ν/dE_ν in Eq. (24) is to be replaced by ϕ_{ν_μ} , ϕ_{ν_τ} or ϕ_{ν_e} (Eq. 15) as the case may be.

From Eqs. (8,9), we see that the unparticle decay of neutrinos depend on the scaling dimension $d_{\mathcal{U}}$ (non integral number) and the neutrino-unparticle coupling strength λ_ν^{ij} . In what follows, both λ_ν^{ij} and just λ both will signify the same coupling. In order study how a possible unparticle decay of neutrinos can affect the UHE neutrino signal from GRB, we vary these parameters and the subsequent variation of the ratio R (Eq. 23) is calculated with the formalism discussed so far. The unparticle decay of neutrinos considered here, $|\nu_2\rangle$ and $|\nu_3\rangle$ are considered unstable and subject to undergo unparticle decay while only $|\nu_1\rangle$ is stable. Therefore, in Eq. (8) we need the masses m_2 and m_3 for unparticle decay of respective neutrinos. In the present calculations the value of m_2 is estimated from $m_2 = \sqrt{\Delta m_{32}^2}$, where $\Delta m_{32}^2 = m_3^2 - m_2^2$ (normal hierarchy) and $\Delta m_{32}^2 = 2.5 \times 10^{-3} \text{ eV}^2$ (from

atmospheric neutrino oscillation) for the present work. The value of m_3 thus follows. The cutoff scale $\Lambda_{\mathcal{U}}$ is taken to be 1 TeV.

Fig. 1 shows the variation of R with coupling strength λ_{ν}^{ij} for three different values of scaling dimension $d_{\mathcal{U}}$ for a neutrinos from a GRB with fixed redshift $z = 0.03$. This redshift corresponds to a baseline length of 3.8×10^{21} Km. The ratio R becomes insensitive to the variation of coupling for higher values of $d_{\mathcal{U}}$. This phenomenon can be understood from Fig. 2 where the variation of decay lifetime τ (in terms of $\frac{\tau}{m}$) with $d_{\mathcal{U}}$ is shown. This plot clearly shows that $\frac{\tau}{m}$ increases with the increase of $d_{\mathcal{U}}$. From Fig. 2, for example, $\frac{\tau}{m} \sim 10^{18}$ when $d_{\mathcal{U}} = 1.3$. For UHE neutrinos with an energy $\sim 10^6$ GeV coming from a GRB at 100 Megaparsec distance ($L \sim 10^{21}$ Km), exponential decay term in Eq. (10) tends to 1 and thus the muon track to shower ratio (R) tends to that for mass flavour oscillation. The value of the ratio R in case of mass flavour oscillation scenario for the same set of oscillation parameters and baseline length of GRB neutrinos ($z = 0.03$) is calculated as $R_{\text{mass-flavour}} = 3.23$ and for no oscillation, R is computed as $R_{\text{no osc}} = 5.6$ for the same GRB.

Since the coupling λ_{ν}^{ij} appears at the denominator of the expression for $\frac{\tau}{m}$ (Eq. 8), for higher values of coupling the decay effects may appear if for those values, the decay length λ_d becomes \sim the baseline length L . This is also apparent in Fig. 1. One also observes from Fig. 1 that for suitable values of scale dimension $d_{\mathcal{U}}$ and the unparticle coupling strength λ , the ratio R can differ from mass flavour value (without the decay) and also from no oscillation value to a considerable extent (in fact the value R can even be very close to 0). Therefore, for certain conditions of unparticle scenario, the unparticle decay of neutrinos may indeed be probed by observing the UHE signal at a kilometer scale detector like IceCube.

In Fig. 3, the variations of the muon track to shower ratio R with $d_{\mathcal{U}}$ are shown for four different values (0.0001, 0.001, 0.01 and 0.1) of couplings λ_{ν}^{ij} . For higher values of $d_{\mathcal{U}}$ and for lower values of λ , decay effects vanish and the value of R tends to that for only mass-flavour oscillation. A fact

that can be understood from Eqs. 8-11 and also discussed above. But once again, Fig. 3 shows substantial variation of the results from those obtained considering only mass-flavour oscillation, for certain ranges of values of $d_{\mathcal{U}}$ and coupling λ_{ν}^{ij} . For example, for $d_{\mathcal{U}} = 1.2$ and $\lambda = 0.001$, the value of $R \simeq 1.27$ — a variation of more than 60% from the mass-flavour oscillation value. Again, to obtain similar effects for higher value of $d_{\mathcal{U}}$, the value of λ is also to be increased. But perhaps, the unparticle coupling with neutrino may not be very high and the scale dimension $d_{\mathcal{U}}$ is to remain within the value $1 < d_{\mathcal{U}} < 2$. This may be mentioned that in Ref. [12] the unparticle decay of neutrinos is used to study the constraints on $d_{\mathcal{U}}$, $\Lambda_{\mathcal{U}}$ etc.

The muon yield and muon to shower ratio are also affected by the GRB neutrino flux. The GRB neutrino flux has a $d^{-2}(z)$ dependence, (Eq. 4) where $d(z)$ is the distance of a GRB with redshift z . The flux also ofcourse has a E^{-2} dependence (Eq. 1). In an event of the unparticle decay of neutrinos, the variation of muon to shower ratio (R) signal at a kilometer scale detector like IceCube for different GRBs at different redshift distances is investigated in Fig. 4. The variations of R with z are plotted for three different values of unparticle coupling ($\lambda = 0.0001, 0.001, 0.01$). Similar variation for pure mass-flavour oscillation and for no oscillation or decay are also shown in Fig. 4. From Fig. 4, one sees that to obtain a variation of $\sim 50\% - 60\%$ for the value of R from that of the pure mass-flavour oscillation value, the coupling is to be large or redshift z (and hence GRB distance $d(z)$) also should be large enough. For the latter case, however, the neutrino flux from GRB will be reduced (Eq. 4) which may affect significant detection.

4 Summary and Discussions

We have considered here the recently proposed “unparticle stuff” associated with the possible existence of a nontrivial scale invariant sector with scale dimension $d_{\mathcal{U}}$. Such “unparticle stuff” appears to be a nonintegral number $d_{\mathcal{U}}$

of invisible particles. Unparticles may open up unexpected phenomenology and in the present work we consider the decay of neutrinos to unparticles. The consequences of such decay process is explored with the example of ultra high energy neutrinos from cosmological Gamma Ray Bursts. We calculate how the unparticle decay of neutrinos, if exists, affects the neutrino fluxes (of all active flavours) from GRB on arriving at earth. We then attempt to estimate how the effect a possible unparticle decay of neutrinos can change the detection yield of GRB neutrinos from only mass-flavour oscillation (or no oscillation) at a kilometer scale ice Cerenkov detector like IceCube. The GRB neutrinos are chosen because of the astronomical baselines of GRB neutrinos (for a terrestrial detector), the oscillatory part is averaged out and one obtains an overall suppression for muon neutrinos. For the present calculation we probe the ratio R of the total secondary muon tracks to the total shower events at the detector for various possible scenarios. The muons and the showers are obtained as the secondary particles following the CC and NC interactions of the cosmic neutrinos with the terrestrial rock and detector material. We obtain more than 60% – 65% deviation of the value of R from only the mass-flavour oscillation scenario for some cases of unparticle decay of neutrinos considered here.

Such deviations can be probed by an efficient detector. The volume of IceCube is 1 Km³ but this is the volume over which the optical modules (OM) are sparsed. Even if a CC interaction vertex occurs in the ice outside this volume, the resulting secondary particle can be detected by IceCube if it enters the specified detector volume. Thus the effective volume of the detector is in fact more than 1 Km³. Although, IceCube detector has better energy and angular resolutions, there are backgrounds from cosmic muons and atmospheric neutrinos. Simulation study of IceCube detector by Ahrens et al [29] indicates that the cosmic neutrino signal from the diffuse cosmic neutrino flux is well below the atmospheric limit. A diffuse cosmic isotropic flux results from the summation of the cosmic sources. But here, we are considering the flux from specific GRBs. Thus they are like point sources. In

this case, one expects an excess of events from a particular directions. This considerably reduces the background if the detector has very good angular resolution. The simulation done at Ref. [29] with 1° angular search cone shows an improvement of the measurement with the diffuse flux. Moreover, the atmospheric neutrino spectrum falls more steeply ($\sim E^{-3.7}$) as against hard GRB neutrino spectrum ($\sim E^{-2}$) from the shock acceleration mechanism considered here. Considering all these, a very large deviation of the signal from the expected ones (e.g. the ones expected from mass-flavour oscillation) may be probed with IceCube. But detailed simulation is required to estimate the deviation of the results that can be probed by the detector. In the present work, the purpose is to study the effects if neutrino decay to unparticles vis-a-vis undergoes mass-flavour oscillation and no oscillation and GRB neutrinos are considered for that purpose.

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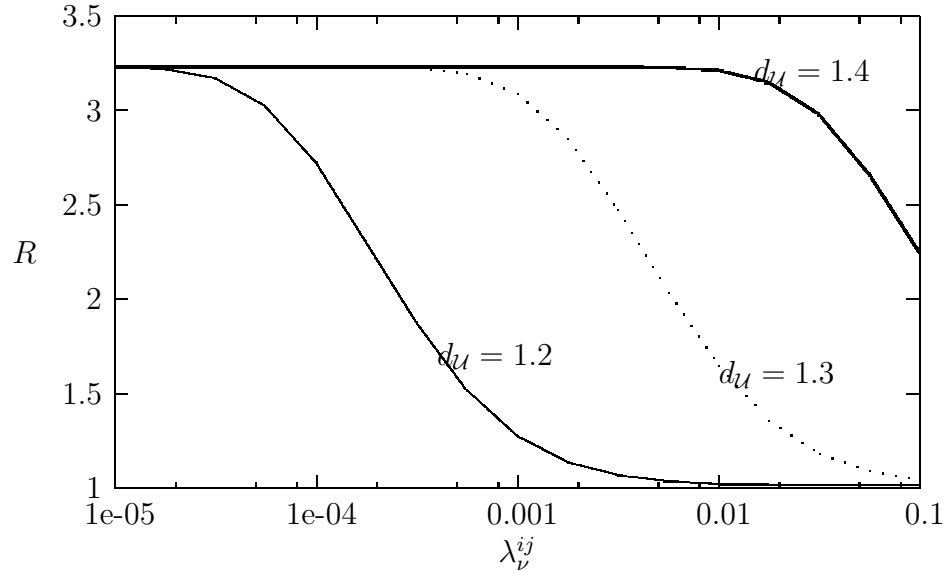


Fig. 1 Variation of R with the unparticle coupling strength λ_ν^{ij} for fixed different values of scaling dimensions d_U

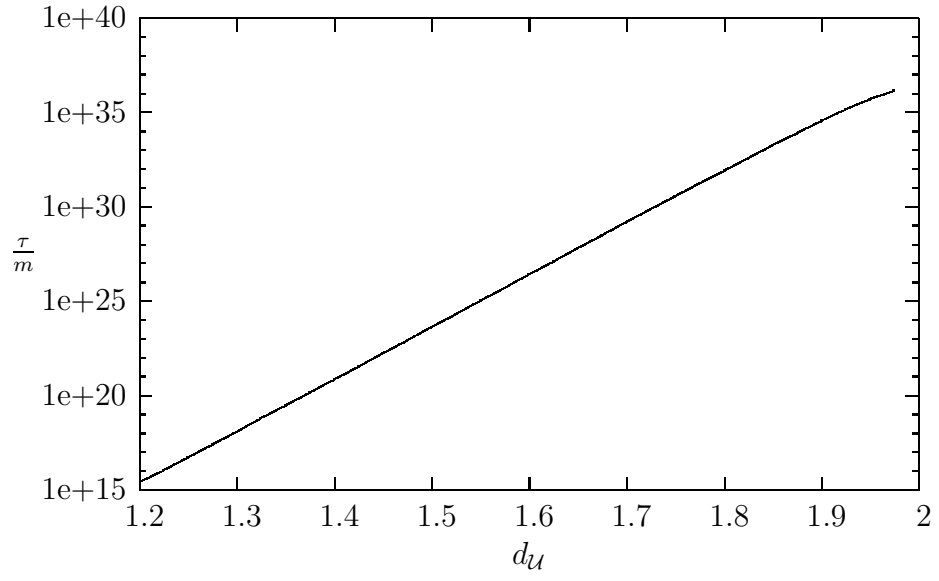


Fig. 2 Variation of τ/m (eV⁻²) with scale dimension d_{\square} . Case shown for $m = m_2$. See text for details.

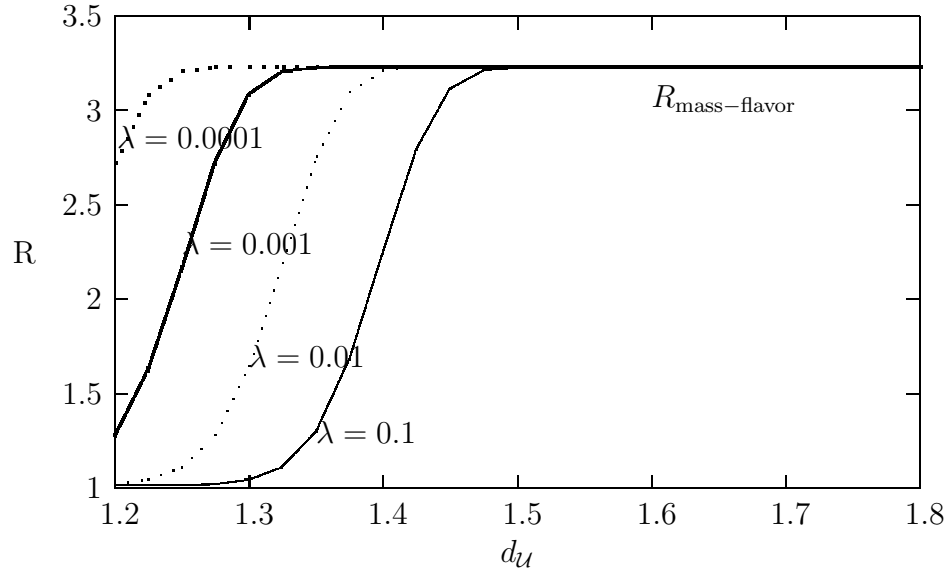


Fig. 3 Variation of R with $d_{\mathcal{U}}$ for different values of coupling. GRB redshift $z = 0.03$. See text for details.

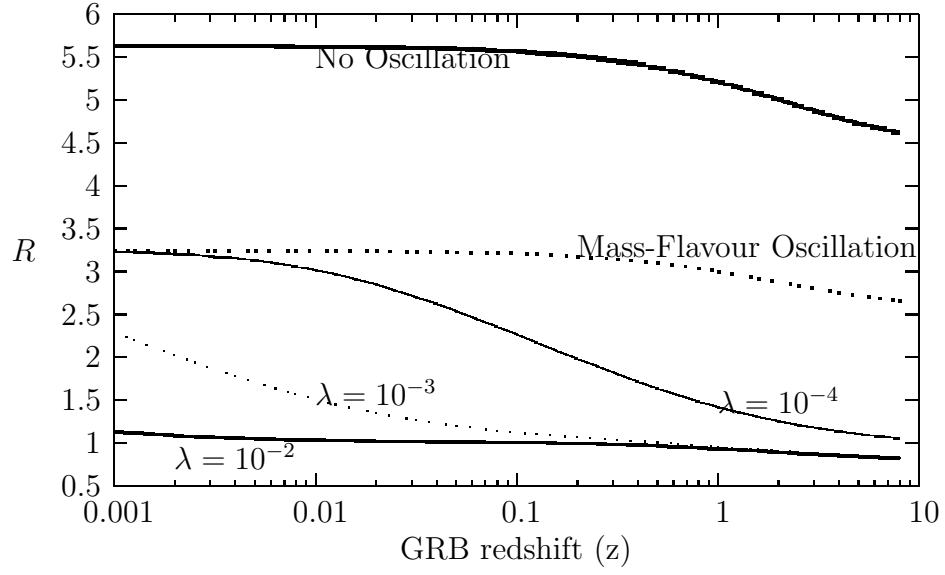


Fig. 4 Variation of ratio R with GRB distance (in terms of redshift z) for unparticle decay for three values of unparticle couplings with $d_{\mathcal{U}} = 1.2$. Also shown are the only mass-flavour oscillation case and no oscillation (no decay) case.